AP Calculus BC 2022 - 2023

Optional Summer Enrichment Assignment

Non-Calculator

Name:__________________________________

Topic 1 Equation of a line

1. Determine the slope of the line that passes through the points (-1, 6) and (11, -6).
2. Find the equation of the line that passes through the point (1, -1) and has a slope of -3. Leave your answer in point-slope form.
3. Find an equation of the line that passes through (-1, -3) parallel to the line $2x+y=19$. Leave your answer in slope-intercept form.
4. Find an equation of the line that passes through (8, 17) and is perpendicular to the line $x+2y=2$. Leave your answer in standard form.

Topic 2 Functions

A. Composition of functions

** Find two functions *f* and *g* such that $h(x)=[f\circ g](x)$. Neither function may be the identity function $y=x$.

1. $h(x)=\sqrt{x^3-4}$
2. $h(x)=\dfrac{1}{x^2-6x+9}$
3. $h(x)=(x+3)^2+5(x+3)+7$

** Evaluate each expression using the values in the table.

1. $(f\circ g)(9)$
2. $(g\circ f)(4)$
3. $(f\circ f)(2)$
4. $(g\circ g)(16)$

x	0	1	2	3	4
f(x)	0	1	4	9	16

x	0	1	4	9	16
g(x)	0	1	2	3	4

** If $f(x)=\dfrac{1}{x}$, $g(x)=x^2$, and $h(x)=\sqrt{x-3}$, find the composition of two functions and state the domain.

1. $g[f(x)]$
2. $g[h(x)]$
3. $f[h(x)]$
4. $f[f(x)]$

B. Inverse Functions

1. If $f(x)=3x^3-1$, find its inverse, $f^{-1}(x)$.
2. Show that $f(x)=e^{x-3}+2$ and $g(x)=\ln(x-2)+3$ are inverses each other.

C. Even and Odd Functions

** Determine if the following functions are even, odd, or neither.

1. $f(x)=-2x^5+3x^3-7x$
2. $f(x)=3x^4-2x^2+5$
3. $f(x)=5x^3+x+2$
4. $f(x)=e^x-e^{-x}$
5. $f(x)=\dfrac{x^2}{x^4+5}$
6. $f(x)=\dfrac{x}{x+1}$

Topic 3 Factor

** Factor completely.

1. $x^4 - 81$
2. $54x^3 + 250y^3$
3. $3x^2 - 36xy + 108y^2$
4. $x^2 + 14x + 49 - 81y^2$
5. $x^3 - xy^2 + x^2y - y^3$
6. $(x-3)^2(2x+1)^3 + (x-3)^3(2x+1)^2$

Topic 4 Solving Polynomial and Rational Equations

1. $7x^2 - 5x = 0$
2. $x^3 - 4x^2 + x + 6 = 0$
3. $(3x-1)^2 = 32$
4. $3x^3 - 24x^2 + 21x = 0$
5. $x^2 - 6x + 1 = 0$
6. $3x^2 - 6x + 2 = 0$
7. $\frac{1}{x-3} - \frac{2}{x+3} = \frac{2x}{x^2-9}$
8. $x + \frac{1}{x} = \frac{13}{6}$

Topic 5 Exponents & Logarithms

** Simplify the expression.

1. $\log_8 \frac{1}{16}$
2. $e^{2\ln 5}$
3. $\frac{\ln 8}{\ln 2}$

** Expand the expression using the property of logarithms.

1. $\log\left[\frac{\sqrt[3]{y}}{x^2 z^{\frac{1}{5}}}\right]$
2. $\ln\left[\frac{5x}{\sqrt{x-7}\,(3x+5)}\right]$

** Condense the expression using the property of logarithms.

1. $\frac{1}{2}\log(x+5) - 2\log x + 3\log(x-2) - 5\log(x+1)$
2. $2\left[\ln(x-1) - 3\ln(x+2) - \frac{1}{3}\ln(x+5)\right]$

** Solve the equation.

1. $\log_8(x-5) = \frac{2}{3}$
2. $\log(5x) + \log(x-1) = 2$
3. $4^{3x} = 8^{x+1}$
4. $5^x = 3e^x$ Leave your answer in exact form.
5. $2e^{-x} - 3 = 11$
6. $3^{5x+1} = 5^{2x-3}$ Leave your answer in exact form.

Topic 6 Transformations of Functions

** Describe the transformations from $f(x)$ to $g(x)$, where $g(x)$ is defined below.

1. $g(x)=f\left(\frac{x}{5}\right)$

2. $g(x)=\frac{1}{7}f(x)$

3. $g(x)=-f(x)$

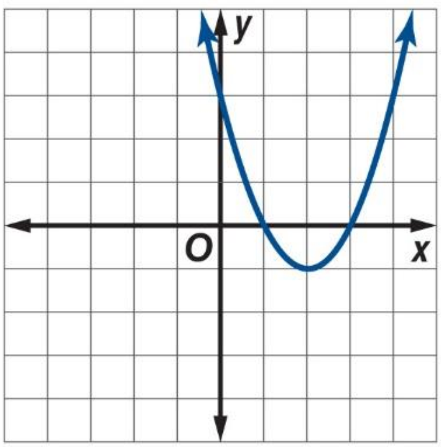
4. $g(x)=f(-x)$

5. $g(x)=9f(x)$

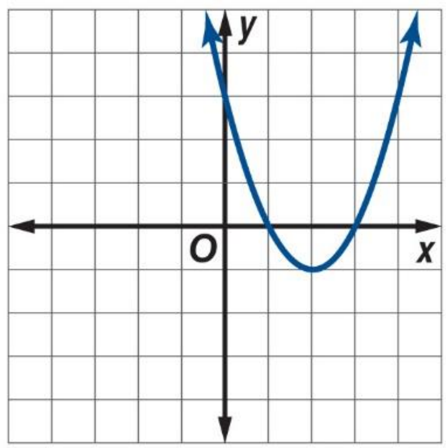
6. $g(x)=f(x-3)+5$

** Sketch the following graph using $f(x)=x^2-4x+3$

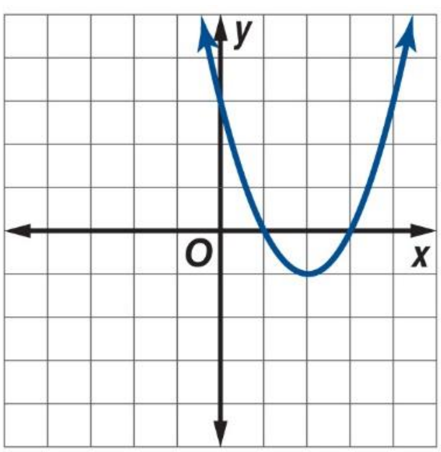
1. $y=f(x+2)-3$



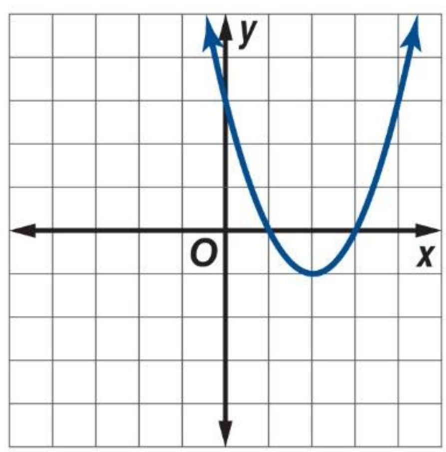
2. $y=f(-x)$



3. $y=|f(x)|$

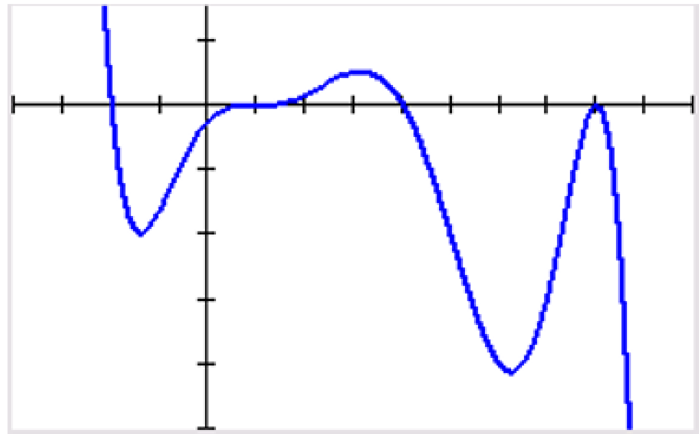


4. $y=f(|x|)$



Topic 7 Function Analysis

1. Use the **complete** graph of a polynomial function $f(x)$ to answer the following: $x \in [-4, 10]$ and x-scale is 1



   A. Is the degree of $f(x)$ even or odd?

   B. Is the leading coefficient of $f(x)$ positive or negative?

   C. What are the **distinct** real zeros of $f(x)$?

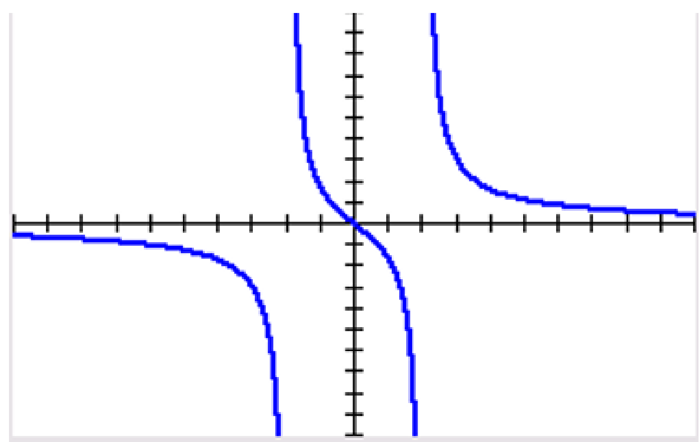
   D. What is the **least** degree of $f(x)$?

   E. How many turning points does it have?

   F. Stationary inflection point occurs at what value of x?

   G. Describe the end behaviors

2. Use the graph of $f(x) = \dfrac{5x}{x^2 - 4}$ to answer the following:



   A. Find the domain

   B. Find the equation of vertical asymptote

   C. Find the equation of the horizontal asymptote

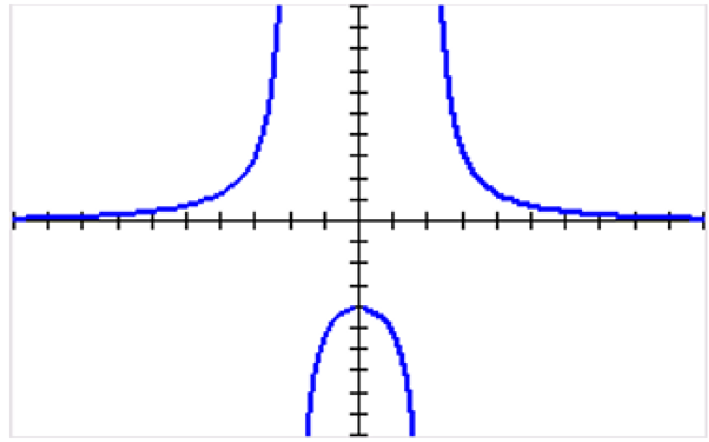
   D. Find the range

   E. Is it an even function or odd function?

   F. Describe the vertical asymptotic behaviors

   $\lim_{x \to -2^-} f(x) =$ _________ $\qquad$ $\lim_{x \to 2^+} f(x) =$ _________

3. Use the graph of $f(x)=\dfrac{16x}{x^3-4x}$ to answer the following:



A. Find the domain.

B. Find the equation of the vertical asymptote.

C. Find equation of the horizontal asymptote.

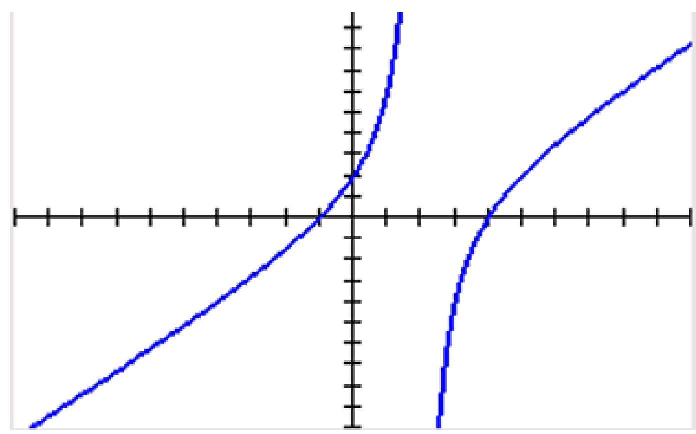
D. Find the range

E. Is it an even function or odd function?

F. Describe the vertical asymptotic behaviors

$\lim_{x\to -2^-} f(x) =$ ________ $\lim_{x\to 2^+} f(x) =$ ________

4. Use the graph of $f(x)=\dfrac{x^2-3x-4}{x-2}$ to answer the following:



A. Find the domain

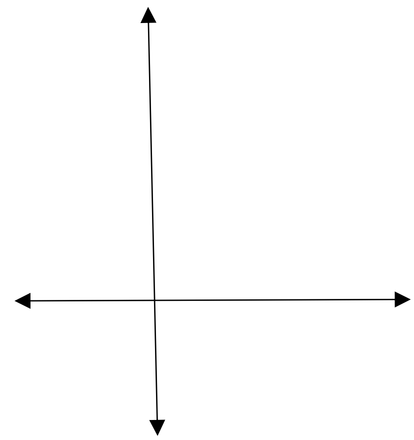
B. Find the equation of the vertical asymptote.

C. Find the x-intercepts and y-intercept

D. Find the equation of the slant asymptote.

E. Find the range.

5. Graph and analyze $f(x) = -2 \cdot e^{-x} + 4$



A. Parent function:

B. Domain:

C. Horizontal Asymptote:

D. Describe the behavior near the horizontal asymptote using the limit notation.

E. Range:

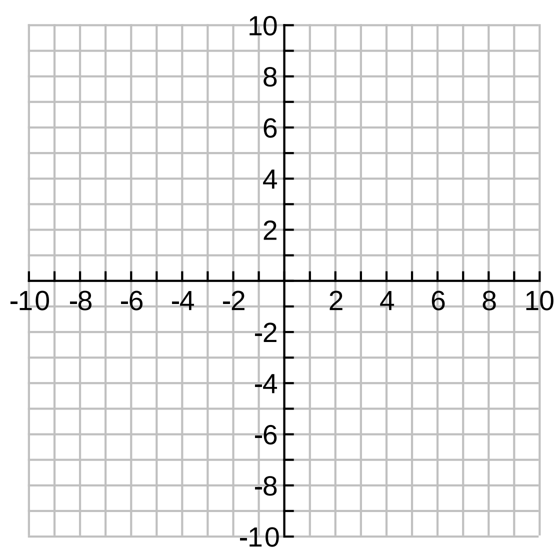
F. Describe the behavior of the function using the concavity.

G. Key points:

H. Graph the function.

Topic 8 Piecewise Functions

** Use $f(x) = \begin{cases} -x, & x \le 3 \\ \frac{2}{3}x - 4, & x > 3 \end{cases}$ to answer the following:



1. Evaluate the function.

A. $f(-3)$ B. $f(0)$

C. $f(3)$ D. $f(6)$

2. Graph the function.

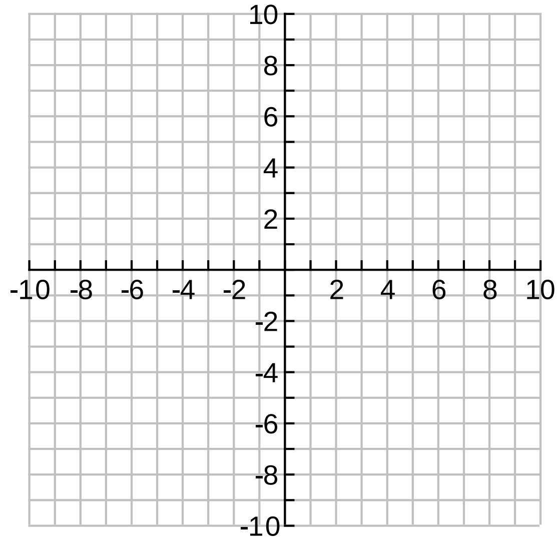
3. Is the function continuous at $x = 3$?

** Use $f\left(x\right)=\begin{cases}-2x-6, & x<-1\\ 2x-2, & x\geq -1\end{cases}$ to answer the following:

1. Evaluate the function.

A. $f\left(-3\right)$ B. $f\left(-1\right)$

C. $f\left(0\right)$ D. $f\left(5\right)$



2. Graph the funciton.

3. Is the function continuous at $x=-1$?

Topic 9 Trigonometry

The following Trigonometric Identities **MUST** be **memorized.**

Reciprocal Identities	**Quotient Identities**	**Pythagorean Identities**
$\sin x=\frac{1}{\csc x}$ $\csc x=\frac{1}{\sin x}$ $\cos x=\frac{1}{\sec x}$ $\sec x=\frac{1}{\cos x}$ $\tan x=\frac{1}{\cot x}$ $\cot x=\frac{1}{\tan x}$	$\tan x=\frac{\sin x}{\cos x}$ $\cot x=\frac{\cos x}{\sin x}$	$\sin^2 x+\cos^2 x=1$ $1+\tan^2 x=\sec^2 x$ $1+\cot^2 x=\csc^2 x$

Co-Function Identities	**Odd / Even Identities**	
	Odd	**Even**
$\sin\left(\frac{\pi}{2}-\theta\right)=\cos\theta$ $\cos\left(\frac{\pi}{2}-\theta\right)=\sin\theta$	$\sin\left(-\theta\right)=-\sin\theta$	
$\csc\left(\frac{\pi}{2}-\theta\right)=\sec\theta$ $\sec\left(\frac{\pi}{2}-\theta\right)=\csc\theta$	$\csc\left(-\theta\right)=-\csc\theta$	$\cos\left(-\theta\right)=\cos\theta$
$\tan\left(\frac{\pi}{2}-\theta\right)=\cot\theta$ $\cot\left(\frac{\pi}{2}-\theta\right)=\tan\theta$	$\tan\left(-\theta\right)=-\tan\theta$	$\sec\left(-\theta\right)=\sec\theta$
	$\cot\left(-\theta\right)=-\cot\theta$	

Double-Angle Identities	**Power-Reducing Identities**
$\sin\left(2x\right)=2\sin x\cdot\cos x$ $\cos\left(2x\right)=\cos^2 x-\sin^2 x$ $=1-2\sin^2 x$ $=2\cos^2 x-1$	$\sin^2 x=\frac{1-\cos 2x}{2}$ $\cos^2 x=\frac{1+\cos 2x}{2}$

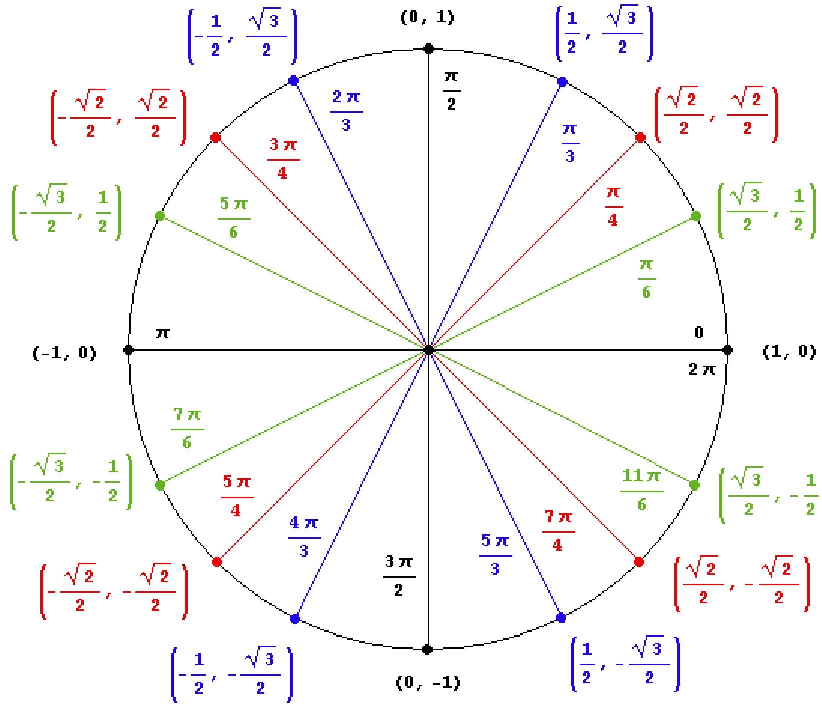
The Radian Measures and Coordinates **Must** be **memorized.**

$\sin\theta = \frac{y}{r}$ $\csc\theta = \frac{r}{y}$

$\cos\theta = \frac{x}{r}$ $\sec\theta = \frac{r}{x}$

$\tan\theta = \frac{y}{x}$ $\cot\theta = \frac{x}{y}$

In a unit circle, $x = \cos\theta$ and $y = \sin\theta$



** Evaluate each expression.

1. $\arcsin\left(-\frac{1}{2}\right)$

2. $\sin^{-1}\left(-\frac{\sqrt{2}}{2}\right)$

3. $\arccos\left(-\frac{\sqrt{3}}{2}\right)$

4. $\cos^{-1}(-1)$

5. $\arctan\left(-\sqrt{3}\right)$

6. $\tan^{-1}(0)$

7. $\cos\left(-\frac{3\pi}{4}\right)$

8. $\sin\left(-\frac{9\pi}{4}\right)$

9. $\tan\left(\frac{5\pi}{6}\right)$

10. $\csc\left(\frac{3\pi}{2}\right)$

11. $\cot(-\pi)$

12. $\sec\left(\frac{\pi}{3}\right)$

** Simplify the expressions.

1. $\sin^2\theta + \cot^2\theta \cdot \sin^2\theta$

2. $\sin^2\left(\frac{\pi}{6}\right) + \cos^2\left(\frac{\pi}{3}\right)$

** Use the Sum and Difference Identities to find the <u>exact</u> value of each trigonometric expression.

$\cos[\alpha+\beta] = \cos\alpha \cdot \cos\beta - \sin\alpha \cdot \sin\beta$	$\cos[\alpha-\beta] = \cos\alpha \cdot \cos\beta + \sin\alpha \cdot \sin\beta$
$\sin[\alpha+\beta] = \sin\alpha \cdot \cos\beta + \cos\alpha \cdot \sin\beta$	$\sin[\alpha-\beta] = \sin\alpha \cdot \cos\beta - \cos\alpha \cdot \sin\beta$

1. $\sin(47°) \cdot \cos(13°) + \cos(47°) \cdot \sin(13°)$

2. $\cos\left(\frac{7\pi}{8}\right) \cdot \cos\left(\frac{5\pi}{24}\right) + \sin\left(\frac{7\pi}{8}\right) \cdot \sin\left(\frac{5\pi}{24}\right)$

3. $\sin(110°) \cdot \cos(80°) - \cos(110°) \cdot \sin(80°)$

4. $\cos\left(\frac{11\pi}{36}\right) \cdot \cos\left(\frac{13\pi}{36}\right) - \sin\left(\frac{11\pi}{36}\right) \cdot \sin\left(\frac{13\pi}{36}\right)$

5. $\cos(75°)$

6. $\sin\left(\frac{\pi}{12}\right)$

** Use $\cos(2\theta) = \cos^2\theta - \sin^2\theta = 1 - 2\sin^2\theta = 2\cos^2\theta - 1$ to derive the Power-Reducing Identities.

1. $\sin^2 x = \frac{1-\cos 2x}{2}$

2. $\cos^2 x = \frac{1+\cos 2x}{2}$

** Find all solutions of the equation in the interval $[0, 2\pi)$

1. $2\cos x - \sqrt{3} = 0$

2. $\sec^2 x = \sec x + 2$

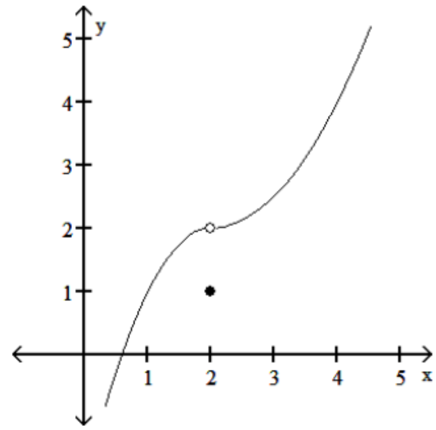
3. $2\sin x \cdot \cos x + \cos x = 0$

4. $\sin x - \sqrt{3}\cos x = 0$

Topic 10 Limits

** Evaluate Limits Graphically.

1. Use the graph of $f(x)$ to answer the following:



A. Evaluate $\lim_{x\to 2^-} f(x)$

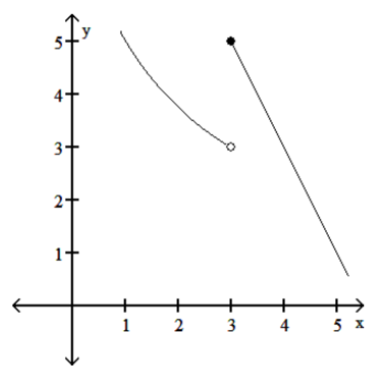
B. Evaluate $\lim_{x\to 2^+} f(x)$

C. Evaluate $\lim_{x\to 2} f(x)$

D. Evaluate $f(2)$

E. Is $f(x)$ continuous at $x = 2$?

2. Use the graph of $f(x)$ to answer the following:



A. Evaluate $\lim_{x\to 3^-} f(x)$

B. Evaluate $\lim_{x\to 3^+} f(x)$

C. Evaluate $\lim_{x\to 3} f(x)$

D. Evaluate $f(3)$

E. Is $f(x)$ continuous at $x = 3$?

** Evaluate Limits Analytically.

1. Find each limit if $\lim_{x\to c} f(x) = 2$ and $\lim_{x\to c} g(x) = -3$

A. $\lim_{x\to c}\sqrt{2f(x)-4g(x)}$

B. $\lim_{x\to c}\left[f(x)+2\right]^3$

C. $\lim_{x\to c}\dfrac{2f(x)+3g(x)}{g(x)-f(x)}$

D. $\lim_{x\to c}\dfrac{\left[f(x)\right]^2}{1-g(x)}$

** Evaluate Limits Algebraically.

1. $\lim_{x\to -3}\left(-2x^2+1\right)$

2. $\lim_{x\to 5}\dfrac{x^2-5x}{x^2-25}$

3. $\lim_{x\to 3}\dfrac{x^2-x-6}{x-3}$

4. $\lim_{x\to 4}\dfrac{x-4}{\sqrt{x}-2}$

5. $\lim_{x\to 2}\dfrac{1-\frac{2}{x}}{x^2-4}$

6. $\lim_{x\to 3}\dfrac{x^3-27}{x-3}$

7. $\lim_{x\to 0}\dfrac{2-\sqrt{4+x}}{x}$

8. $\lim_{x\to 1}\dfrac{(x-1)^4}{x^4-1}$

9. $\lim_{x\to -\infty}\dfrac{x^2+2}{2x^2+5}$

10. $\lim_{x\to \infty}\dfrac{2x^2+3x}{5x^4-2x+1}$

11. $\lim_{x\to \infty}\dfrac{1-x^2}{1+x}$

12. $\lim_{x\to \infty}\dfrac{3000x^3}{x-1000x^3}$

** Continuity

1. Find the value of k that makes the piecewise function $f(x)$ be continuous.

$$f(x)=\begin{cases} kx^2-1 & \text{for } x>-1 \\ -2x+3 & \text{for } x\le -1 \end{cases}$$

2. Write down the piecewise function so that $f(x)=\dfrac{\sin x}{x}$ is continuous for all real numbers.

Topic 11 Sequences and Series

1. Write a rule for the nth term of the sequence $8, \frac{8}{3}, \frac{8}{9}, \frac{8}{27}, ...$ and find a_7

2. Write a rule for the nth term of the sequence $-2, 4, -8, 16, ...$ and find a_{10}

3. Use sigma (summation) notation to write $1-8+27-64+125-.......$

4. Use sigma (summation) notation to write $-1+4-9+16-25+.......$

5. Use sigma (summation) notation to write and evaluate the sum: $\frac{2}{3}-2+6-...+486$

6. Use sigma (summation) notation to write and evaluate the sum: $2+\frac{1}{2}+\frac{1}{8}+......+\frac{1}{2048}$

7. Determine whether each infinite geometric series converges or diverges. If it converges, find the sum.

A. $\sum_{n=0}^{\infty}\left[3\left(\frac{1}{2}\right)^n\right]$ B. $\sum_{n=0}^{\infty}\left[2\left(\frac{5}{3}\right)^n\right]$ C. $\sum_{n=1}^{\infty}\left[3\left(-\frac{2}{3}\right)^{n-1}\right]$ D. $\sum_{n=1}^{\infty}\left[9\left(-\frac{2}{5}\right)^n\right]$

Topic 12 Parametric Equations

**** Parametric Forms of Rectangular Equations**

A. Line through (x_1, y_1) and (x_2, y_2)

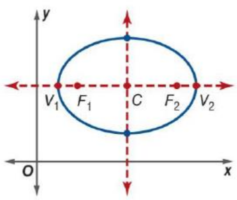
$$\begin{cases} x = x_1 + t(x_2 - x_1) \\ y = y_1 + t(y_2 - y_1) \end{cases}$$

B. Circle: Center (h, k) radius $= r$

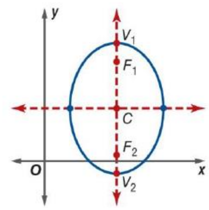
$$\begin{cases} x = h + r\cos\theta \\ y = k + r\sin\theta \end{cases}$$

C. Ellipse: Center (h, k), $a > b$

$$\begin{cases} x = h + a\cos\theta \\ y = k + b\sin\theta \end{cases} \Leftrightarrow \frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$$

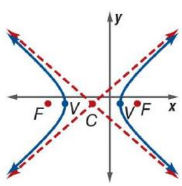


$$\begin{cases} x = h + b\cos\theta \\ y = k + a\sin\theta \end{cases} \Leftrightarrow \frac{(x-h)^2}{b^2} + \frac{(y-k)^2}{a^2} = 1$$

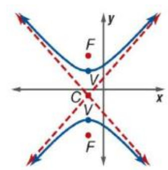


D. Hyperbola: Center (h, k), $\sec^2 t - \tan^2 t = 1$

$$\begin{cases} x = h + a\sec\theta \\ y = k + b\tan\theta \end{cases} \Leftrightarrow \frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$$



$$\begin{cases} x = h + b\tan\theta \\ y = k + a\sec\theta \end{cases} \Leftrightarrow \frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$$



1. Find a parametric representation for the line segment through $(-2, 5)$ and $(4, -13)$.

2. Find a parametric representation of the circle $(x-2)^2 + (y+3)^2 = 25$

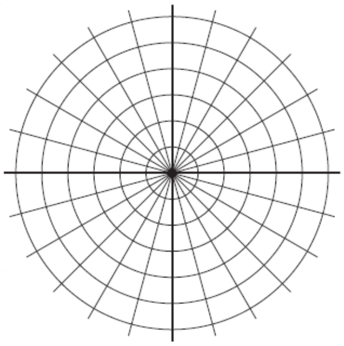
3. Find a parametric representation of $\frac{(x+1)^2}{36} + \frac{(y-4)^2}{16} = 1$

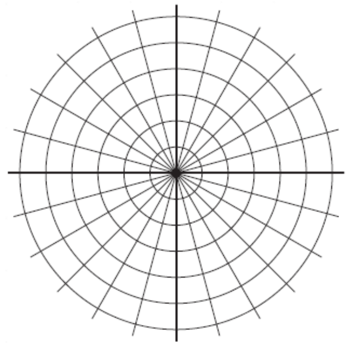
4. Find a parametric representation of $\frac{(y-2)^2}{4} - \frac{(x+3)^2}{9} = 1$

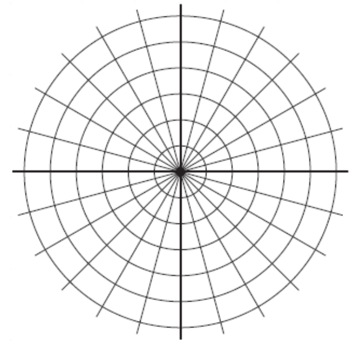
Topic 13 Polar Coordinates

1. A. Graph the following points in polar coordinates: $A\left(1,\frac{5\pi}{4}\right)$, $B\left(-1,\frac{7\pi}{4}\right)$, and $C\left(2,-\frac{3\pi}{2}\right)$.

B. Give three alternative representations for each point, where $-2\pi < \theta < 2\pi$







2. Convert Polar Coordinate to Rectangular Coordinate.

A. $\left(6,\frac{2\pi}{3}\right)$ B. $\left(-3,-\frac{7\pi}{4}\right)$ C. $\left(-2,-\frac{5\pi}{6}\right)$ D. $\left(5,\frac{5\pi}{3}\right)$

3. Convert Rectangular Coordinate to Polar Coordinates (Four different representations, where $-2\pi < \theta \le 2\pi$)

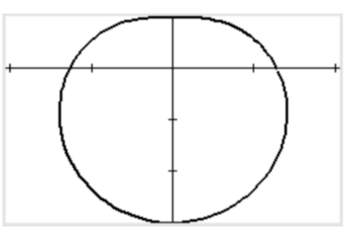
A. $\left(-\sqrt{3},1\right)$ B. $\left(\frac{\sqrt{2}}{2},-\frac{\sqrt{2}}{2}\right)$ C. $(0,-5)$ D. $(-6,0)$

4. Describe the graph of each polar equation. Confirm each description by converting to a rectangular equation.

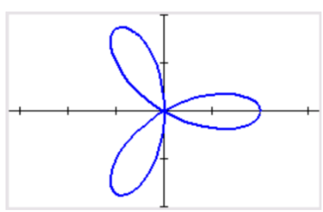
A. $\theta=\frac{\pi}{3}$ B. $r=\sec\theta$ C. $r=-4\cos\theta$ D. $r=6\sin\theta$

5. Match each graph with one of the given equations.

A.



B.



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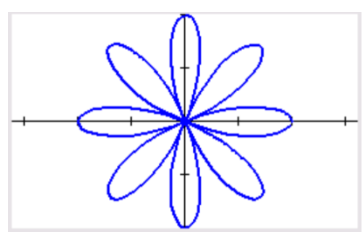
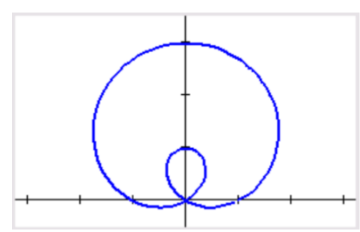


D.



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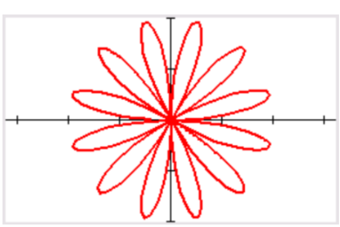

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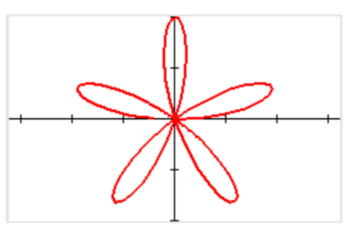
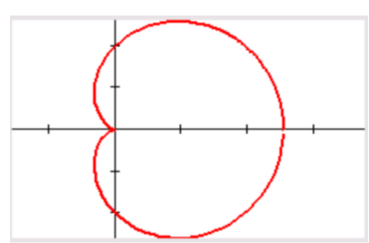
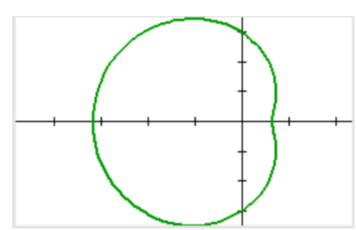


G.



H.



I. $r=2\cos(3\theta)$ II. $r=2-\sin\theta$ III. $r=2\sin(5\theta)$ IV. $r=1+2\sin\theta$

V. $r=2+2\cos\theta$ VI. $r=2\cos(4\theta)$ VII. $r=3-2\cos\theta$ VIII. $r=2\sin(6\theta)$